Proposed ideas on time travel and the possibility of causality violation.

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Abstract- The puzzle of resolving a time travel method has been a bizarre to both theoretical and applied physics fields. At first, time was absolute in terms of Newtonian principles. But then, Einstein had come with his evolutionary ideas that have opened the door for multiple contributions to be observed and considered. Despite the proposed mathematical possibility for traveling 'relatively' to the future, the application for that is still a hardcore till this day. Multiple scientists had a belief in the idea of transportation between different time intervals at any desired instant, however, it remains non-practical in terms of reality case-scenarios. In this extensive literature review, a pick of distinguishable theories for the case of time travel is studied.

Index Terms- Causality violation – Field Equation – General relativity – Rotating Cylinder – Spinning Universe – Wormhole

I. INTRODUCTION

Time travel is one of the most phenomenal hypotheses that has made scientists became questioned and has taken most of their time to study and search for its possibility. In the past, it was proposed by Sir Isaac Newton that time is absolute; it doesn't change with any kind of influence from anybody, which means that time is the same in any place across the horizons of the universe. This had been the case for eras until the time of proposal of the special theory of relativity in 1905 by Albert Einstein. The theory has discussed two effects that are resulted from the upgrading speed of a moving object. One of them results in a rate of change of time passage, also known as the time dilation. Time dilation works under the conditions of relativistic superior velocities that approaches the speed of light. The equation is given as the following:

$$\Delta t_{moving} = \frac{\Delta t_{rest}}{\sqrt{1 - \frac{u^2}{c^2}}} \quad [1]$$

Time dilation was the first-proposed key that made a step towards approaching the concept of time travel. Although it represents a mathematical illustration for a possibility of time travel, there are two flaws apparent when understanding that approach. The first flaw is clear: the law is hypothetical, which means that there are not practical applications to prove it in real life. This flaw lies behind the fact that nothing travels through space faster than light – as it is said by Einstein. The second conflict is mathematically obvious: it is only possible to travel 'relatively' to the future. There has no apparent explanation for traveling back to the past – as time cannot move backwards. For this reason, multiple contributions were made to propose other more efficient hypotheses for a more-free time travel between past, present, and future.

A manifold of models and hypotheses were presented by scientists to represent a practical assumption for a time machine and both its physical principles and mechanism. The first model of a time machine involves a 'wormhole'. The second one was an infinitely long, rotating cylinder. The final one included a visualization of a 'spinning universe' [2]. Each one of these models will be discussed in detail with their pros and cons.

II. BODY

A. 1st Model: Wormholes

Under the constraints of theoretical physics, time travel through a wormhole is hypothetically possible; nevertheless, we can only ever travel backward. In a blog post for Forbes, astronomer Ethan Siegel explains how, given the parameters of Einstein's General Relativity, a person may go back in time through a wormhole. The wormhole, he added, is the starting point, since it is a space doorway formed by the reversal of the direction of energy fluctuations. Each of the distinct fluctuations would generate a space with a curvature that opposes the other. If these two were then connected, a wormhole would result. If it lasted long enough, a particle could theoretically pass through. [3]

To scale this up enough that a human could pass through would be more challenging. It would necessitate the initial discovery of particles with negative mass and energy. After achieving this, it would be required to create a supermassive black hole and its negative mass/energy counterpart. This, he claims, should make a traversable wormhole possible. [3]

The approach for the investigation begins with defining the Einstein-Cartan theory for the simplest explanation of general relativity. The action integral at first is given by

$$S = \int d^4x\sqrt{-g}\,\{\frac{-1}{2\varsigma}\left(\tilde{R} + 2\Lambda\right) + \mathcal{L}_m\}$$

Where $\kappa = 8\pi G/c4$ being the gravitational coupling constant, $\tilde{R}$ being the Ricci curvature scalar, Λ is the cosmological constant and is $\mathcal{L}_m$ the Lagrangian of the matter fields.
Substituting several factors, and varying the action equation with respect to an indicated metric gives the equation of Einstein-Cartan theory:

$$G_{\mu\beta}\left(\{\}\right) - \Lambda g_{\mu\nu} = \kappa\left(T_{\mu\beta} + \theta_{\mu\beta}\right)$$

The second term on the right-hand side of the equation, represents a correction to the dynamical energy-momentum tensor which considers the spin contributions to the geometry of the spacetime. This gives the appropriate opening to explain how a wormhole must be traversable to either validate the proposed theory of time travel or disapprove it. the presence of a spinning fluid is considered when explaining the wormhole geometries. Such a fluid is known as Weyssenhoff fluid, which is considered as a continuous macroscopic medium whose microscopic elements are composed of spinning particles. [4]

We begin by proposing the equation of line of symmetry of a rotating black hole that is filled with Weyssenhoff fluid:

$$ds^2 = -e^{2\Phi(r)}dt^2 + \left(1 - \frac{b(r)}{r}\right)^{-1} dr^2 + r^2 d\Omega^2$$

where dΩ2 is the standard line element on a unit two-sphere; φ(r) and b(r) are redshift and shape functions respectively.
For the wormhole to be traversable, one must demand that there are no horizons present. So, φ(r) should be finite everywhere so that there is no singularity and event horizon in spacetime. The field equations with using these 4-vector coordinates:

$$u_\mu = [e - \varphi(r), 0, 0, 0]$$

$$v_\mu = \left[0, \sqrt{1 - \frac{b(r)}{r}}, 0, 0\right]$$

This leads to the following:

$$\begin{aligned}
\rho(r) &= \frac{1}{4r^2}\left[4b'(r)+r^2S^2(r)-4\Lambda r^2\right],\\
p_r(r) &= \frac{1}{4r^3}\left[8\phi'(r)(r^2-rb(r))-4b(r)+S^2(r)r^3+4\Lambda r^3\right],\\
p_t(r) &= \frac{1}{4r^3}\Big[4r^2\phi''(r-b(r))+4r^2\phi'^2(r-b(r))-2r\phi'(rb'(r)-2r+b(r))\\
&+ S^2(r)r^3-2rb'(r)+4\Lambda r^3+2b(r)\Big],
\end{aligned}$$

Figure 1: parameters of a traversable wormhole with spinning fluids

ρ is the energy spinning density, while p_r and p_t are the radial and tangential components of the spinning pressure across the wormhole [4].

For wormhole energy parameters, the following equations are:

$$\begin{aligned}
\rho(r)+p_r(r) &= \frac{1}{2r^3}\left[4r\phi'(r)(r-b(r))+S^2(r)r^3+2rb'(r)-2b(r)\right],\\
\rho(r)+p_t(r) &= \frac{1}{2r^3}\Big[2r^2\phi''(r)(r-b(r))+2r^2(r-b(r))\phi'^2(r)\\
&- r\phi'(r)\left(rb'(r)-2r+b(r)\right)+S^2(r)r^3+rb'(r)+b(r)\Big].
\end{aligned}$$

If it is possible to travel across the wormhole, then we would travel with the speed of light or even faster. By the effect of time dilation, we estimate that we have traveled to a later, future time.

B. 2nd Model: Rotating Cylinder

An idea has been identified by Van Stockum, stating that there is a solution for the equations of Einstein, which proposes the idea of causality violation (another term for breaking the constrains of time to make time travel feasible). The hypothesis has visualized an infinite-rotating cylinder with a significant gravitational influence. This gravitational field performs a supernatural violation in the causality of time, meaning that time intervals might collapse on each other, creating what might be considered as a closed-timeline intervals.
In research done by Frank J. Tipler with his companions, the ideas of Stockum had been proposed and explained. The construction of the theory began with the metric for the field of cylinder in which the centrifugal forces are balanced by the gravitational attraction:

$$ds^2 = H(dr^2+dz^2)+Ld\varphi^2+2Md\varphi dt-Fdt^2$$

Where ‘z’ measures distance along the cylinder axis ($-\infty < z < \infty$)[4, rotating cylinders, and the possibility of global causality violation]. According to the theory, the phenomenon of causality violation would be achievable if an interior field (usually has a matter in rest state) had been linked to an exterior field. The equations for the interior field are:

$$H = e^{-a^2r^2}$$
$$L = r^2(1-a^2r^2)$$

$$\rho = 4a^2e^{a^2r^2}$$
$$M = ar^2$$
$$F = 1$$

Where ‘a’ is the angular velocity of the cylinder, ‘r’ is the radial distance from the axis ($0 < r < \infty$), ‘ρ’ is the matter density and ‘M’ is the moment inertia of the cylinder on the center of mass of its axis. [5]
To validate the hypothesized theory, Stockum developed a procedure to come up with an exterior solution to achieve the speed of light in the product of ‘aR = 1’ (in the units determined by the researcher). The developed exterior solutions for 3 cases are as follows:

1- For $0 < aR < \frac{1}{2}$:

$$H = e^{-a^2r^2}\left(\frac{r}{R}\right)^{-2a^2R^2}$$

$$L = \frac{R.r.\sinh(3\epsilon+\theta)}{2sinh.2\epsilon.cosh\epsilon}$$

$$M = \frac{r.\sinh(\epsilon+\theta)}{sinh2\epsilon}$$

$$F = \frac{r.\sinh(\epsilon-\theta)}{R.sinh\epsilon}$$

With $\theta = (1-4a^2R^2)^{\frac{1}{2}}.\ln\left(\frac{r}{R}\right),\ \epsilon = \tanh^{-1}(1-4a^2R^2)^{\frac{1}{2}}$

2- For $aR = \frac{1}{2}$:

$$H = e^{-\frac{1}{4}}\left(\frac{r}{R}\right)^{-\frac{1}{2}}$$

$$L = \frac{1}{4}Rr\left[3+\ln\left(\frac{r}{R}\right)\right]$$

$$M = \frac{1}{2}r\left[1+\ln\left(\frac{r}{R}\right)\right]$$

$$F = \left(\frac{r}{R}\right)\left[1-\ln\left(\frac{r}{R}\right)\right]$$

3- For $aR > \frac{1}{2}$:

$$H = e^{-a^2r^2}\left(\frac{r}{R}\right)^{-2a^2R^2}$$

$$L = \frac{R.r.\sinh(3\beta+\gamma)}{2sinh.2\epsilon.cosh\epsilon}$$

$$M = \frac{r.\sinh(\beta+\gamma)}{sinh2\epsilon}$$

$$F = \frac{r.\sinh(\beta-\gamma)}{R.sinh\epsilon}$$

With $\gamma = (4a^2R^2-1)^{1\backslash 2}\ln\left(\frac{r}{R}\right),\ \beta = \tan^{-1}(4a^2R^2-1)^{1\backslash 2}$

It has been stated that the causality violation occurs at for the metric conditions at ‘aR > ½’. To avoid the problems caused by sinusoid factors, the equations must be transformed in the form of:

$$t' = At + B\varphi, \qquad \varphi = Ct + D\varphi, \qquad A, B, C, D\ are\ constants$$

Because of the limitations imposed by a number of elements, it is speculated that these circumstances may not be applicable to cylinders that represent the real world. Because of this, it has been suggested that it may be possible to apply the criteria to a long, finite spinning cylinder rather than an infinite one. This is because of the fact that the length of the cylinder would be limited. A Kerr field would be produced as a result of the cylinder being shrunken into a ring and the process being reversed. The fact that this field contains lines that behave like closed-timelike loops is essential to understanding the concept of causality violation. It is speculated that there is a portion of this approximately sized field in which the factor g_ takes on a negative value, which would imply the existence of a violation. [5].

C. 3rd Model: Spinning Universe

Kurt Gödel proposed a new solution to the Einstein equations that differed from all previous solutions in two crucial points. First, it simulated a revolving universe. This revealed for the first time that general relativity does not fully account for "Mach's principle." It is quite unexpected considering that Mach's principle served as one of the driving forces behind Einstein's development of general relativity. It also made it possible for closed, time-like curves to exist (unlike the other solutions). In actuality, this curve cuts across each and every point in this space-time. As a result, is has been feasible for an observer to go into the past. [6]
The explanation of solution has started with defining the corresponding rotation of the vector field υ^{μ}, that is, ω^{μ}:

$$\omega^{\mu} = \frac{1}{12\sqrt{-g}}\,\epsilon^{\mu ijk}a_{ijk}$$

Where $a_{ijk} = \left(v_i, v_j, v_k\right)\cdot\ \left(\left(\frac{\partial}{\partial xi}, \frac{\partial}{\partial xj}\frac{\partial}{\partial xk}\right)\right)\times\ \left(v_i, v_j, v_k\right)$
Identifying that the coordinate system is geodesic, and by supposing that matter is at rest at the origin in this frame (i.e., the velocity vector for matter is (1,0,0,0) in these coordinates). Then ‘ω’ and its components is obtainable by the formula:

$$\omega^1 = \frac{1}{2}\left(\frac{\partial v^3}{\partial x^2} - \frac{\partial v^2}{\partial x^3}\right) = \frac{1}{2}\left(\frac{\partial}{\partial x^2}\frac{v^3}{v^4} - \frac{\partial}{\partial x^3}\frac{v^2}{v^4}\right)$$

$$\omega^0 = 0$$

The angular velocity in Newtonian physics is represented by the first three components, and zero represents the fourth. It appears that the only vector deriving from ‘v’ that fits with the classical idea in these coordinates is ‘w’, therefore it makes sense to define this as rotation in space-time. [6]
Unlike the universe, which is principally built by Einstein’s space-time curvature, Godel universe exhibits unique features in both its matter distribution and preferred motion. Godel universe is homogenous with

constant matter distribution, meaning that each point in the space-time is identical (according to relativity principles). As a result, a preferred rotational motion can be identified, making the solution time orientable but pathological in the sense that there is no universal concept of time and that time-like closed loops, or potential paths that always move "ahead" in time yet intersect themselves, actually exist.
The solution of Godel lies in the manifold (M, g_ab), where M = R^4, and the metric g is given by (in the canonical coordinates for R^4, which we will call rectangular coordinates):

$$ds^2 = a^2(dx_0^2 - dx_1^2 + \frac{e^{2x_1}}{2} dx_2^2 - dx_3^2 + 2e^{x_1}dx_0dx_2)$$

a > 0 is constant. The matrix describing the geometrical coordinates of $g_{\mu\nu}$ and its inverse are illustrated below:

$$g_{\mu\nu} = a^2 \begin{pmatrix} 1 & 0 & e^{x_1} & 0 \\ 0 & -1 & 0 & 0 \\ e^{x_1} & 0 & \frac{1}{2}e^{2x_1} & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}, \quad g^{\mu\nu} = \frac{1}{a^2} \begin{pmatrix} -1 & 0 & 2e^{-x_1} & 0 \\ 0 & -1 & 0 & 0 \\ 2e^{-x_1} & 0 & -2e^{-2x_1} & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}.$$

Figure 2: Figure 2: the matrix that describes the geometrical aspect of Godel Universe

On the following:

$$e^{x_1} = cosh(2r) + cos(\phi)sinh(2r)$$
$$x_2e^{x_1} = \sqrt{2}sin(\phi)sinh(2r)$$
$$tan(\frac{\phi}{2} + \frac{x_0 - 2t}{2\sqrt{2}}) = tan(\frac{\phi}{2})e^{-2r}$$
$$y = 2x_3$$

for r ≥ 0, 0 ≤ φ ≤ 2π. The 2π periodicity of φ and association of r with distance suggests that we call these cylindrical coordinates for M. The metric in this coordinate system becomes

$$ds^2 = 4a^2(dt^2 - dr^2 - dy^2 + (\sinh^4 r - \sinh^2 r)d\varphi^2 + 2\sqrt{2}\sinh^2 r d\varphi dt)$$

The matter in the Godel solution is dust with constant density and 4-velocity u^μ, so, its stress-energy-momentum tensor is $T_{\mu\nu} = \rho u^\mu u^\nu$. Finally, the solution given has negative cosmological constant, which corresponds to a positive pressure. [5]
After several calculations, the solution proposed The manifold (M,g) solves the field equations for dust with constant density ρ = 1/8πGa^2, (where G is Newton's gravitational constant) and cosmological constant Λ = −1/2a^2. The solution involved 3 unique properties:

1- M is homogeneous. That is, for any two events A and B in M, there is a transformation of M carrying A into B. Furthermore, M has rotational symmetry, meaning for every point A of M, there exists a one parameter group of transformations of M carrying A into itself.
2- M is rotating everywhere with velocity $2\sqrt{(\pi G\rho)}$, where ρ is the matter density, and G is Newton's Gravitational constant.
3- M is time orientable. However, M has closed time-like curves, and furthermore, any two points of this space-time can be connected by a time-like loop. Therefore, no global time coordinate exists for M.

Godel universe proposed the most feasible solution among other solutions as it is based on simple mathematical calculations made with logical assumptions [6].

III. DISCUISSION: TIME TRAVEL PARADOX

Perhaps mathematics proved the possibility behind creating a time machine. But if it is possible to travel back in the past to change the future, would the same future still exist? If it has been changed, what would have been the purpose of going in the first place? The famous grandfather paradox serves as an illustration that time travel is not conceivable.

The grandfather paradox, which occurs in some time travel scenarios, is exemplified by the impossibility of someone travelling back in time to kill their grandfather (who could no longer go on to create one's parent, so where does that leave you and your ancestor-killing event?). Suppose you own a time machine that enables you to travel back in time. You mistakenly kill one of your grandparents or other direct ancestors before they have children. That would change the entire course of future events, including your own birth, which would no longer occur. However, if you were not born in the future, you could not murder your ancestor in the past; hence, the dilemma.

Stephen Hawking, possibly the most masterful physicist in recent history, was uniquely qualified to discuss the reality of time travel. He presented a lecture on "space and time warps" in 1999, demonstrating how Einstein's theory of general relativity might enable time travel by bending space-time inward. Theoretically, a particular type of wormhole may enable time travel and, thus, the opportunity to eliminate a crucial ancestor. However, as Hawking noted in his presentation, a wormhole might be able to loop back to a previous period of time; this scenario is referred to as a "closed time-like curve" (CTC).

Nevertheless, if physics permits time travel in reverse, wouldn't the grandfather paradox still be problematic? In this case, Hawking proposed two solutions to circumvent the contradiction. The "consistent histories" model is the first, according to which all of time—past, present, and future—is strictly preset. According to this model, you can only go back in time if you have already been there in your own history. [7]

The logical paradox has given researchers a headache, in part because "closed timelike curves" are possible, according to Einstein's theory of general relativity. This would theoretically allow an observer to travel back in time and interact with themselves from the past, which could endanger their own existence. According to the findings of these experts, however, such a contradiction would not have to arise because the events in question would adjust themselves.

Consider the case of patient zero with the coronavirus. "You might try to block patient zero from becoming infected, but in doing so, you would catch the virus and become patient zero, or someone else would," Tobar said in an interview with the university's news agency. "Patient zero" refers to the individual who has not yet been affected by the virus. In other words, a time traveler could make changes, but the original outcome would still find a way to happen, perhaps not in the same way that it happened in the first timeline, but close enough so that the time traveler would still exist and would still be motivated to go back in time. [Case in point] Tobar stated that "the significant occurrences would just re-adjust themselves around you no matter what you did." [8]

IV. CONCLUSION

In terms of practicability, time travel is still a concept that cannot be approached. Theoretical physicists carry on with the necessary in-depth investigation to get closer to discovering the one and only answer that will allow them to triumph over the challenges of time. Each solution contained logical statements as a point of strength, but there was no practical application because the ability was not viable. This was a flaw of each approach.

It suggested highly certain criteria for both the pressure and the density for the wormhole solution. However, it appears that the necessary energy does not exist. This is quite sad. Due to the enormous volumes that would be required, there is no known form of energy that would function effectively.
In spite of the fact that it has produced logical and detailed requirements and shown various situations, the solution of an eternally revolving cylinder appears to be peculiar when seen from a geometrical perspective.
The spinning universe solution, in contrast to the other two solutions, eliminates the possibility of closed-timelike curves by linking all of the points on the spacetime curve to one another. This opens the door for the possibility of going to different points in the past. The only problem with the solution is that it cannot be used to solve the problem in our universe because it does not rotate.
There are inconsistencies between the mathematical model and the practical application of the potential of building time machines. The case will, however, continue to be discussed, and researchers will keep working to find a method to make it more accessible..

V. References

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